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OPTIMAL SOURCE DISTRIBUTION FOR MAXIMUM POWER DISSIPATION AT THE CENTER OF A LOSSY SPHERE

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ABSTRACT

The ideal penetration limits for localized, non-invasive heating of tumors at the center of a volume of muscle tissue are determined. Using both an integral formulation and a modal approach, the optimum surface phase and amplitude source distributions which prevent excessive heating of healthy, intervening tissue are derived.

INTRODUCTION

In non-invasive microwave hyperthermia cancer therapy, it is important to know the penetration depth limits of radiation which produces local power maxima. For treatment which provides heat at depth at the site of a localized tumor, overheating intervening tissue must be avoided. Two questions are vital to understanding the possiblities and limitations of this type of treatment: "What is the maximum radius of a sphere of biological tissue for which an optimally distributed source will generate as much power at its center as at its surface?" and "What is this optimum source distribution?"

LOSSY SPHERE FIELD SOLUTIONS

A spherical geometry allows the greatest exposure of a focal target point to sources on the surface for a given minimum depth of lossy medium. Thus the sphere represents the best possible non-invasive hyperthermia configuration. Although medical applications of heating spherical volumes are limited to only head and whole body, the knowledge gained from studying this best-case heating geometry will aid in the design of more practical hyperthermia systems.

The development of the optimal solution uses both the surface-current integration formula and the spherical harmonic solutions to the wave equation.

The greatest constructive interference at the center of a sphere results when the polarizations of all the surface sources are parallel: pointing in, say, the z-direction, as shown in Figure 1. Any additional symmetrical radial component (or, correspondingly, polar-angle component) ends up cancelling itself in the center, and any unsymmetrical components perpendicular to z obviously do not contribute to the z-component.

Integrating these parallel currents on a spherical surface of radius R is straightforward for a uniform distribution $\mathbf{J}(\mathbf{r}') = \delta(r-R)\,\hat{\mathbf{z}}$.. Without loss of generality, choose observation points along the z-axis, which lie a distance r from the sphere's center. Using the law of cosines for the source to observer distance in the Green's function integral[1] yields:

$$E = -j\omega(\bar{\bar{I}} + \frac{1}{k^2}\nabla\nabla)\cdot\int_0^{\pi} d\theta' \int_0^{2\pi} d\phi' R^2 sin\theta' \mu \frac{e^{-jk\sqrt{R^2+r^2-2rRcos\theta'}}}{4\pi\sqrt{R^2+r^2-2rRcos\theta'}}\hat{\mathbf{z}} \qquad (1)$$

The resulting power in a sphere of muscle tissue, normalized to that at the center, is plotted as a function of radius at $\theta = \pi/2$ in Figure 2 for some of the important hyperthermia frequencies. For these plots, $k = \beta - j\alpha$, and the values of α and β were obtained for the various frequencies using experimentally derived values of dielectric constant and conductivity[2]. It is the intersections with unity that determine the maximum allowable radius of tissue that can be heated without overheating the surface.

MODAL ANALYSIS

Although the uniform surface current distribution intuitively seems optimal, additional improvement becomes apparent from a modal viewpoint. The harmonics of a sphere produce electric field as represented by[2]:

$$E = \hat{r}\ A_n \frac{n(n+1)}{kr} j_n(kr)\, P_n(cos\theta) \quad (2)$$

$$+ \hat{\theta}\ A_n\, [j_{n-1}(kr) - \frac{n}{kr} j_n(kr)]\ \ [n\ cos\theta P_n\,(cos\theta) - nP_{n-1}(cos\theta)]\ \ (\frac{1}{sin\theta})$$

The key feature in this equation is that since the spherical Bessel functions, j_n, vary as $(kr)^n$, the only mode which contributes to field in the center, r = 0, is the n = 1 mode. The Legendre polynomials $P_0(x)$ and $P_1(x)$ evaluate to 1 and x respectively, and so it becomes evident that the first mode corresponds to the uniform surface current case. However, since the higher order modes approach 0 in the center, they can be used to counteract the large, undesirable values of field elsewhere. Specifically, a distribution can be synthesized from modes with appropriate chosen phase and amplitude to partially cancel the field at the surface and thereby increase the maximum allowable sphere size.

The distribution of power on the surface of a large, lossy sphere for uniform current varies as $sin^2\theta$, as seen from equation (1) with n = 1 and by recalling $j_1(kR) << j_0(kR)$ for $kR >> 1$. Reducing the surface peak at $\pi/2$ is accomplished by adding the n = 2 and the n = 3 modes, which contain $sin(3\theta)$ and $sin(5\theta)$ terms, such that the surface power (rather than current) is more nearly a uniform function of θ. With the object of minimizing the maximum surface value of the sum of modes, the coefficients B_1 and B_2 of the function $sin(\theta)+B_1 sin(3\theta)+B_2 sin(5\theta)$ which produce 3 equal peaks are sought An iterative method is used to find the solutions to this transendental equation, which results in B_1 = .2355, B_2 = .0640. Additional, higher order terms could be used, but the reduction in power would only be in the order of .005, not warranting the added computational complexity.

Combining the first three modes using equation (2) with the A_n chosen to normal the Bessel function values at R, to combine the nth-order Legendre polynomial values, and to normalize the the power at the center, results in the surface power distribution shown in Figure 3. Plotted as a function of θ, it is observed that there is a sizeable reduction of peak power and that the power is more evenly spread across the surface. Also, it is clear that the fifth order ripple is very close to ideal. The normalized maximum surface power is lowered by a factor of 0 78. Figure 4 plots the power as a function of radius at $\theta = \pi/2$ for the sum of 3 modes, for the same frequencies as in figure (2). Comparing these two figures shows maximum radius increases of 1.72, 0.84, 0.57, and 0.32 cm for frequencies of 100, 433, 915, and 2450 MHz, respectively.

CONCLUSION

The dimensions of the largest convex volume of muscle tissue which can be heated non-invasively, without overheating the surface, has been determined for the standard electromagnetic hyperthermia frequencies. These limits are the theoretical best cases (within 0.5%): it is not possible to improve on them by altering the surface phase or amplitude distribution. For other tissue geometries, the maximum penetration depth will, of course, be lower

Although penetration depth increases with decreasing frequency below 433 MHz., the resolution of the focal spot at the center decreases. However, due to the non-linear dependence of complex dielectric constant on frequency, increasing the frequency does yield an increase in penetration depth for a limited range, as shown by the plot of 915 MHz. power curves. For 433 MHz. $\alpha/\beta = 0.396$, whereas for 915 MHz. it is 0 231. There is a small advantage to using a more uniform power surface distribution than the uniform current ditribution. The improvements are more pronounced for the lower frequencies, since wavelengths are longer, and the slopes of the power curves are shallower.

REFERENCES

1. Kong, J. A., *Electromagnetic Wave Theory,* Wiley, New York, 1986, p. 229.

2. Guy, A. W., "Analyses of Electromagnetic Fields Induced in Biological Tissue by Thermographic Studies on Equivalent Phantom Models," *IEEE Transactions on Microwave Theory and Techniques*, vol MTT-19, no. 2, February 1971, pp. 205-214.

3. Stratton, J. A., *Electromagnetic Theory*, McGraw-Hill, New York, 1941, p. 416.

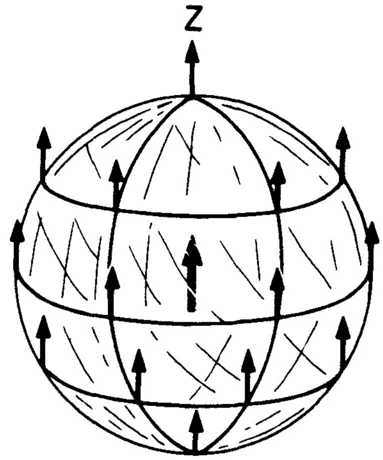

FIGURE 1. Currents polarized in the z-direction on the surface of a sphere, and the resulting maximum constructive interference of electric field at the center.

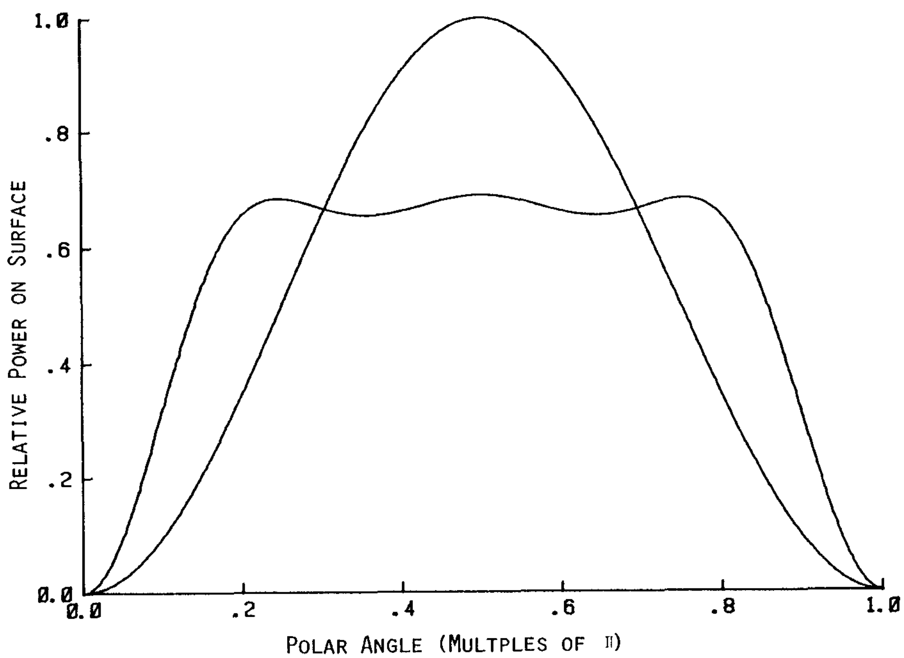

FIGURE 3. Surface power as a function of θ for single mode (uniform current), and three mode (approximate uniform power) distributions.

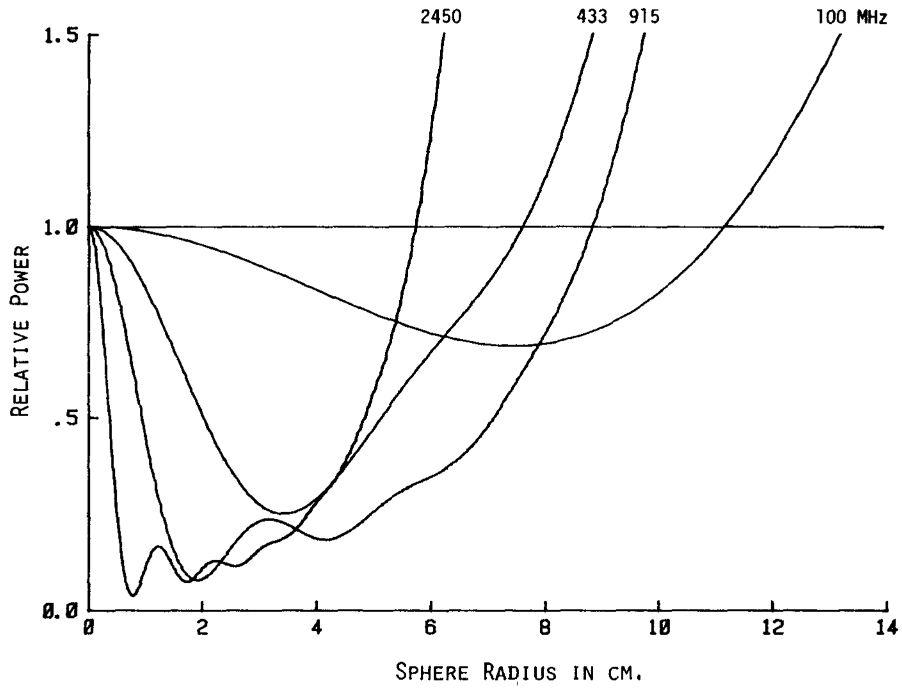

FIGURE 2. Dissipated power in a sphere of muscle tissue as a function of radius for four standard hyperthermia frequencies: uniform current distribution.

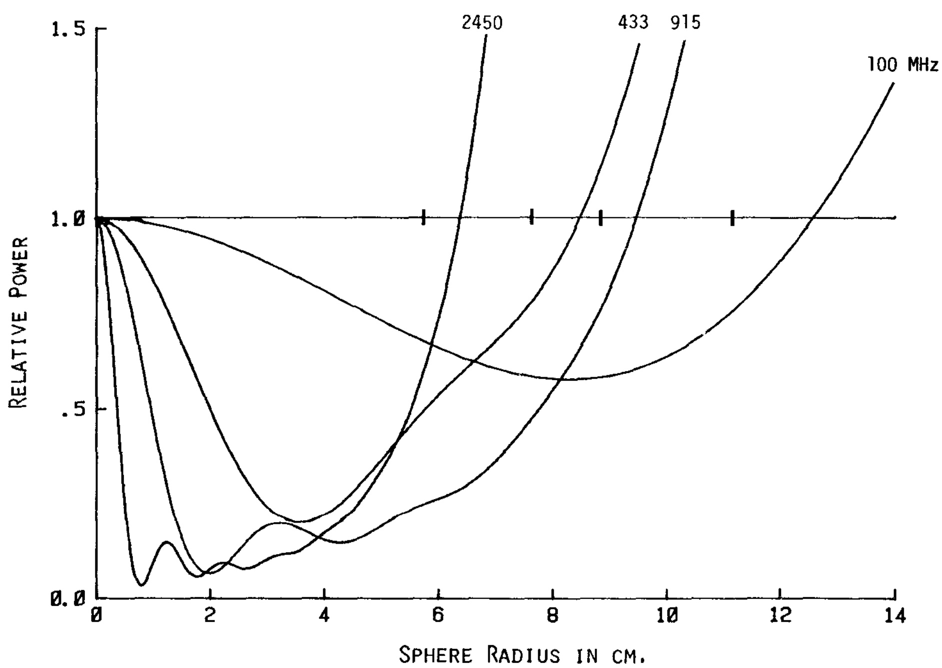

FIGURE 4. Power in a sphere of muscle tissue for approximate uniform surface power distribution.